Physical Layer Models for PAP02: Wireless Communications for the Smart Grid - Task 6

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1 Overview

This document describes computational functions that can be used for coverage analysis and modeling of wireless transmission error probabilities. The transmission error probability (e.g., packet error probability or frame error probability) is modeled using a signal-to-noise ratio (SNR) threshold model. In other words, the probability of transmission error (or communication link outage) is equal to the probability that the received SNR falls below a threshold (i.e., the cumulative distribution of the received SNR). The SNR is modeled as random due to the wireless channel propagation effects of lognormal shadowing and Nakagami-$m$ fading.

Two functions are defined: $\text{SNRcdf()}$ and $\text{SNRcdfCell()}$. They differ in that $\text{SNRcdf()}$ takes as input the nominal SNR (in the absence of fading/shadowing) while $\text{SNRcdfCell()}$ takes coverage radii as input and averages the probability over the coverage area. In translating distances to received SNR, the latter function uses a dual-slope path loss model.

These functions have been tested with MATLAB releases 2007b, 2008a and 2009a, and GNU Octave 3.2.3.

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2 Usage of SNRcdf()

This function is called as

$$\text{SNRcdf}((E_b/N_0)_{rx}, (E_b/N_0)_{req}, \sigma, m)$$

and returns a number between 0 and 1 representing the SNR threshold probability (1) in Section 4.1.

The input parameters and their units are as follows:

- $(E_b/N_0)_{rx}$: nominal received $E_b/N_0$ in the absence of fading or shadowing (dB)
- $(E_b/N_0)_{req}$: minimum required $E_b/N_0$ at the receiver to successfully decode a packet (dB)
- $\sigma$: standard deviation of lognormal shadowing (dB)
- $m$: Nakagami-m fading parameter ($m \geq 0.5$; $m = 1$ corresponds to Rayleigh fading; $m \to \infty$ corresponds to no fading)

3 Usage of SNRcdfCell()

This function is called as

$$\text{SNRcdfCell}(R_{min}, R_{max}, (E_b/N_0)_{req}, R_b, \sigma, m, n_1, flag)$$

and returns a number between 0 and 1 representing the average threshold probability in the cell given as (4) in Section 4.2.

The input parameters and their units are as follows:
Figure 1: Depiction of a cell for coverage analysis

- $R_{\text{min}}$: inner radius of coverage (i.e., the minimum distance a terminal can be located from the BS - Base Station / AP - Access Point) (m)
- $R_{\text{max}}$: outer (maximum) radius of coverage under consideration (m)
- $(E_b/N_0)_{\text{req}}$: minimum required $E_b/N_0$ at the receiver to successfully decode a packet (dB)
- $R_b$: physical layer information data rate (b/s)
- $\sigma$: standard deviation of lognormal shadowing (dB)
- $m$: Nakagami-$m$ fading parameter ($m \geq 0.5$; $m = 1$ corresponds to Rayleigh fading; $m \to \infty$ corresponds to no fading)
- $n_1$: path loss exponent after the breakpoint distance (see Section 4.2.2)
- $\text{flag}$: console output is suppressed if $\text{flag} = \text{true}$ (optional)

Fig. 1 illustrates a cell with a BS/AP, a terminal, and key topology parameters.

Additional parameters are defined at the beginning of the m-file and are listed below with their default values. These values can be changed by directly editing the m-file. Following is the excerpt of code which sets these parameter values.
fc=2.4e9; % carrier frequency (Hz)
eirp_dbm=20; % effective isotropic radiated power (dBm)
d1=10; % breakpoint distance of dual-slope path loss model (m)
n0=2; % path loss exponent before breakpoint
Gr_dBi=5; % receive antenna gain (dBi)
systemLoss_db=1.5; % system loss (dB)
T0=293; % temperature (K)
F_db=5; % noise figure (dB)

4 Calculation of Probabilities

4.1 Probability that Received $E_b/N_0$ is Less Than Required $E_b/N_0$

The probability that the received SNR, $(E_b/N_0)_{rx}$, is less than the required SNR at the receiver, $(E_b/N_0)_{req}$, is evaluated in $\text{SNRcdf}()$ as

$$
\Pr \left[ \left( \frac{E_b}{N_0} \right)_{rx, \text{dB}} < \left( \frac{E_b}{N_0} \right)_{\text{req, dB}} \right]
$$

$$
= \Pr \left[ \left( \frac{E_b}{N_0} \right)_{rx, \text{dB}} + X \left( \frac{E_b}{N_0} \right)_{\text{req, dB}} < \left( \frac{E_b}{N_0} \right)_{\text{req, dB}} \right]
$$

$$
= \Pr \left[ X < \left( \frac{E_b}{N_0} \right)_{\text{req, dB}} - \left( \frac{E_b}{N_0} \right)_{rx, \text{dB}} \right]
$$

$$
= F_X \left[ \left( \frac{E_b}{N_0} \right)_{\text{req, dB}} - \left( \frac{E_b}{N_0} \right)_{rx, \text{dB}} \right]
$$

where $(E_b/N_0)_{rx, \text{dB}}$ is the nominal received $E_b/N_0$ (i.e., in the absence of shadowing and fading), $X = X_{s, \text{dB}} + X_{f, \text{dB}}$ is the combined random attenuation due to shadowing and fading, and $F_X(x)$ is the cumulative distribution function (cdf) of the random variable $X$.

The lognormal shadowing attenuation $X_{s, \text{dB}}$ is modeled as a zero-mean Gaussian random variable with standard deviation $\sigma$. In the presence of lognormal shadowing alone (i.e., $X = X_{s, \text{dB}}$), the cdf of $X$ is

$$
F_X(x) = 1 - \frac{1}{2} \text{erfc} \left( \frac{x}{\sigma \sqrt{2}} \right)
$$

where erfc$(x)$ is the complementary error function.
Based on the assumption of Nakagami-\(m\) fading, the fading attenuation \(X_f\) is modeled as a unit-mean Gamma-distributed random variable. In the presence of fading alone (i.e., \(X = X_{f,\text{dB}} = 10\log_{10} X_f\)), the cdf of \(X\) is

\[
F_X(x) = P(mx, m)
\]

where \(P(x, a) = \frac{1}{\Gamma(a)} \int_0^x e^{-t} t^{a-1} dt\) is the incomplete gamma function (lower regularized).

In the presence of combined lognormal shadowing and Nakagami-\(m\) fading (\(X = X_{s,\text{dB}} + X_{f,\text{dB}}\)), the cdf of \(X\) is evaluated numerically as

\[
F_X(x) = E_{X_f} \left[ F_{X_{s,\text{dB}}} \left( x - 10\log_{10} X_f \right) \right] = \int_0^\infty \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{x - 10\log_{10} y}{\sigma\sqrt{2}} \right) \right] f_{X_f}(y) dy
\]

where the probability density function of \(X_f\) is

\[
f_{X_f}(y) = \frac{m^m}{\Gamma(m)} y^{m-1} e^{-my}
\]

and \(\Gamma(\cdot)\) is the Gamma function.

### 4.2 Average Probability over the Coverage Area

To average probability (1) over the entire coverage area, we must first express (2) as a function of distance and then evaluate its average over the area of the cell. The cdf (2) is a function of distance through the nominal received SNR, that is,

\[
P(d) = F_X \left[ \left( \frac{E_b}{N_0} \right)_{\text{req,dB}} - \left( \frac{E_b}{N_0} \right)_{\text{rx,dB}}(d) \right]
\]

(3)

where \(d\) is the transmitter-receiver distance.

For uniformly distributed terminal locations in the coverage area of Fig. 1, it can be shown that the average of this probability over the cell is given by [1]

\[
\mathcal{P} = \frac{1}{R_{\text{max}}^2 - R_{\text{min}}^2} \int_{R_{\text{min}}}^{R_{\text{max}}} P(r) 2\pi r \, dr.
\]

(4)

This integral is evaluated numerically in \texttt{SNRcdfCell}().
4.2.1 Nominal Received $E_b/N_0$ as a Function of Distance

The nominal received $E_b/N_0$ at distance $d$ in (3) is evaluated as

$$\left(\frac{E_b}{N_0}\right)_{rx,\text{dB}}(d) = EIRP_{\text{dBm}} + G_{r,\text{dBi}}$$

$$-PL_{dB}(d) - L_{s,\text{dB}} - N_{0,\text{dBm/Hz}} - R_{b,\text{dB-b/s}}$$

where

- $EIRP_{\text{dBm}}$ is the effective isotropic radiated power from the transmitter (dBm)
  (set as described in Section 3),
- $G_{r,\text{dBi}}$ is the receiver antenna gain (dBi),
- $PL_{dB}(d)$ is the path loss with distance (dB) (defined in Section 4.2.2),
- $L_{s,\text{dB}}$ is the cumulative system loss due to cabling or other implementation losses (dB),
- $N_{0,\text{dBm/Hz}}$ is the power spectral density of the noise (dBm/Hz), computed as
  $30 + \log_{10}(kT_0F)$ where $k$ is Boltzmann’s constant (J/K), $T_0$ is the
  temperature (K), and $F$ is the receiver’s noise figure, and
- $R_{b,\text{dB-b/s}}$ is the physical layer information data rate (dB-b/s).

4.2.2 Path Loss with Distance

The term for path loss with distance in (5) is calculated using a dual-slope path loss model
and is given as

$$PL_{dB}(d) = PL_0 + \begin{cases} 
10n_0 \log_{10} d, & d \leq d_1 \\
10n_0 \log_{10} d_1 + 10n_1 \log_{10} \left(\frac{d}{d_1}\right), & d > d_1
\end{cases}$$

where $n_0$ and $n_1$ are the path loss exponents of the dual-slope model, $d_1$ is the breakpoint
distance (m) from path loss exponent $n_0$ to $n_1$, and $PL_0$ is the reference path loss at 1 m
given as

$$PL_0 = 20 \log_{10} \left(\frac{4\pi f_c}{c}\right)$$

where $f_c$ is the carrier frequency (Hz) and $c$ is the speed of light (m/s).
5 Sample Results

Fig. 2 plots sample results for the average probability that the received SNR is below a threshold, as a function of the maximum coverage radius, $R_{\text{max}}$, and for four different values of the fading parameter, $m$. These results were generated using `SNRcdfCell()` with the default values in Section 3 and the following input values:

- $R_{\text{min}} = 10$ m
- $R_{\text{max}}$ ranging from 10 m to 1000 m
- $(E_b/N_0)_{\text{req}} = 10.4$ dB
- $R_b = 10^6$ b/s
- $\sigma = 8$ dB
- $m = m \in \{0.5, 1, 2, \infty\}$
- $n_1 = 4$

The curve for $m \to \infty$ corresponds to a channel with lognormal shadowing only (no fading).

References